10 Stellated Polyhedra and Duality

Themes	Duality, stellation, types of polyhedra.
Vocabulary	Stellation, stellated, dipyramid, prism, reflection , rotation.
Synopsis	Make stellated versions of polyhedra and connect coplanar vertices with tape to show the dual polyhedron. Compare symmetries of a polyhedron and its dual.

Overall structure	**Previous**	**Extension**
1 Use, Safety and the Rhombus		
2 Strips and Tunnels		
3 Pyramids (at basic level)	**X**	
4 Regular Polyhedra (some relationships between dual regular polygons)	**X**	
5 Symmetry	**X**	
6 Colour Patterns (part 9 relates the octahedron to the tetrahedron by partial stellation)		**X**
7 Space Fillers		
8 Double Edge Length Tetrahedron (can provide the double edge length tetrahedron which can be extended to the stella octangula in this activity)		
9 Stella Octangula (is an example of what is done here OR can be done as a direct continuation of this activity, if it has not already been done)	**X**	**X**
10 Stellated Polyhedra and Duality		
11 Faces and Edges		
12 Angle Deficit		
13 Torus		

Layout

The activity description is in this font, with possible speech or actions as follows:

Suggested instructor speech is shown here with

possible student responses shown here.
'Alternative responses are shown in quotation marks'.

1 Introduction to stellation by example

If students have already completed **9 The Stella Octangula** then this example is of interest in introducing dipyramids and their duals.

You will need 8 baseless triangular pyramids. For example tell students:

> In pairs take three triangles and tie them together to form the sides of a triangular based pyramid. Put your pyramids without a base in the centre of the class.

Now place two of these pyramids base to base to give a six sided polyhedron, a dipyramid, and tie the laces on the 3 mid points of the edges. See figure 1.

Hold the shape up and explain:

> This is a triangular dipyramid, which means a polyhedron made of two triangular based pyramids. The prefix 'Di' means two.

Figure 1 The triangular dipyramid

> We are now going to stellate this dipyramid by putting another pyramid on each face.
>
> How many faces does the dipyramid have?

6

> How many pyramids do the need to add?

Put three pyramids without bases together as in figure 2.

We will tie these three pyramids now like this, and one other group take another three of the pyramids already made and tie them together the same way.

Figure 2 Three pyramids tied together

When both sets are tied (see figure 2) place one over the top of the dipyramid (see figure 3) and indicate by pointing and touching, while saying:

Each of the three top faces of the dipyramid now has a new pyramid coming out of it.

Figure 3 Stellating the top half of a dipyramid

Now place the other set of pyramids on the bottom half and push the two sets of pyramids together **without** tying yet (figure 4) and again show say:

> Each of the six faces of the dipyramid now has a new pyramid coming out of it. The new shape is called the 'stellated dipyramid'.
>
> These three dipyramid faces that are sloping up have pyramids with these apexes on top, and these three dipyramid faces facing down now have these three pyramids with these apexes below.

Figure 4 Stellation around the dipyramid

After pausing to ensure everyone could see, say:

> Now we know what it looks like, we will take the original dipyramid out before we tie it.

Remove the dipyramid and reassemble the pyramids as in figure 5 and tie together.

Figure 5 The stellated triangular dipyramid

2 Comparing the dipyramid with its stellation

Place the original dipyramid next to the stellation in the same orientation as it would have inside the stellation, see figure 6. Touch each face on the dipyramid and ask:

Show us which pyramid and its apex on the stellated dipyramid corresponds to this one.

That one

Go and touch the one you mean.

Figure 6 Triangular dipyramid with stellation

At this point you may wish to let students build and stellate more shapes. You may wish to continue the directed activity on taping and even duality and then allow them to build and stellate more shapes.

3 Taping and the triangular prism

Figure 7 How to tape between apexes

Provide masking tape and demonstrate taping from one pyramid apex to another as in figure 7:

We can connect the apexes of the new pyramids on the stellated dipyramid with masking tape like this. First wrap it round one apex, then run it to wrap around a nearby one.

What shape do you think this will make if we continue over the all the apexes?

'A triangle', 'verticals', 'a box'

Two people come forward and continue taping.

Do we do on the floor as well?

Yes

Figure 8 All apexes connected by tape

When it is finished ask:

What is the name of the shape made by tape?

A triangular box

What kinds of face does it have?

Rectangles and triangles

How many rectangles?

3

How many triangles?

2

Is there anything special about the triangular faces?

They are equilateral

Anything else?

'horizontal', 'parallel'

What about the rectangles?

Vertical

When the vertical sides are all rectangles and the top and bottom are the same what do we call the shape?

Prism

Yes we call it a prism.

Give the definition of prism in your curriculum, such as

"A prism is defined as a polyhedron with two parallel congruent faces. One of these faces is called the base. Each edge on the base is connected to corresponding edge on the parallel face by a single rectangle perpendicular to the base."

What kind of prism is this?

A triangular prism

Yes, what shape is the base?

A triangle

As the base is a triangle we call it a triangular prism. The prism is named after the shape of the base.

4 Duality

Repeat the steps in section 2 to demonstrate duality. Compare each face of the dipyramid held in the orientation of the internal dipyramid (as it was in figure 6), but this time, if possible, held upright as in figure 8.

Touch a face of the dipyramid and ask:

Which apex on the stellation corresponds to this face on the dipyramid?

This one

Touch it.

What can we call that point on the triangular prism?

A vertex

Now which vertex on the prism corresponds to this face on the dipyramid?

This one.

Does every vertex on the prism correspond to a face on the dipyramid?

Yes

Touch a vertex on the triangular prism and ask:

Which face on the dipyramid does this vertex correspond to?

This one

Touch it

Does every face on the dipyramid correspond to a vertex on the prism?

Yes

So the faces of the dipyramid correspond to vertices of the prism.

What might the faces of the prism correspond to on the dipyramid?

Edges

Show us

No, the vertices

Show us

Run your hand around the tape outlining a triangular face of the prism.

What shape is this face?

A triangle

Show us which vertex on the dipyramid it corresponds to?

How many faces on the dipyramid come together at that vertex?

3

How many vertices does the triangle have?

3

Run your hand around the tape outlining a rectangular face of the prism.

What shape is this face and how many vertices does it have?

Rectangle , 4 vertices

Which vertex on the dipyramid does it correspond to and how many faces meet at that vertex?

This one, 4 faces

Right.

So we call these two shapes, the triangular dipyramid and the triangular prism dual polyhedra.

A polyhedron and its dual have vertices of one corresponding to faces of the other, and vice versa.

How many faces does the triangular prism have?

5

How many vertices does the triangular dipyramid have?

5

The same number.

How many faces does the triangular dipyramid have?

6

How many vertices does the triangular prism have?

6

So this checks out, the faces of one correspond to vertices of the other. They have to be the same number.

Also when two polyhedra are duals we can say that each polyhedron is the dual of the other. Also we can construct the dual of a polyhedron by stellating it and connecting the apexes with tape. The apexes will the vertices of the dual and the tape will be edges of the dual polyhedron.

An interesting question:

How would you stellate a square face?

Use a square based pyramid

How many faces of that pyramid would meet at the apex?

4

5 Compare Symmetries

Ask the students:

Someone find a symmetry of the triangular dipyramid.

Reflection

Show us the reflection

Now someone else see if you can identify a corresponding reflection on the stellation if the two polyhedra are in the same orientation.

Through the middle

Show us which way the shapes are oriented and which way the reflection goes.

Any see any other symmetries?

Altogether both shapes have

- one 1/3 turn rotational symmetry
- three ½ turn rotational symmetries
- four planes of reflection.

Can anyone say why a polyhedron and its dual might have the same symmetries?

Because the only difference is the stellation and it's the same shape underneath

That is right: stellation does not change the symmetries, because all faces are stellated the same way.

6 Other dual polyhedra

Here is a summary list of simple polyhedra and their duals:

- Tetrahedron - tetrahedron

- Cube – Octahedron
- Octahedron - Cube
- Icosahedron - Dodecahedron
- Dodecahedron - Icosahedron
- Dipyramid - Prism (this category includes the cube and octahedron)
- Prism - Dipyramid

This list illustrates that duals come in pairs or in the case of the tetrahedron, the dual of a tetrahedron is a tetrahedron. This is because the dual of a dual is the original polyhedron.

Figure 9 shows a stellated icosahedron, and in tape, its dual dodecahedron. It is assembled from 20 sets of three triangles each.

Figure 9 The stellated icosahedron

