1 Introduction: Use, Safety and the Rhombus

Themes	Terminology, etymology (word origin), 2-D shapes, shape properties, proof by demonstration, symmetry.
Vocabulary	Equilateral, equiangular, rhombus, congruent, parallel, obtuse, isosceles, perpendicular, line of reflection, centre of rotation, rotational symmetry, reflection symmetry.
Synopsis	Short activity to introduce the triangles to a new group of learners, show how to connect them together, review basic geometric vocabulary and possibly explore properties of the rhombus. Finally there is an option to examine two dimensional symmetry, reflection and rotation, for the rhombus and the regular *n*-gons.

Overall structure	**Previous**	**Extension**
1 Use, Safety and the Rhombus		
2 Strips and Tunnels extends basic building and shape		**X**
3 Pyramids extends towards regular polyhedra		**X**
4 Regular Polyhedra		
5 Symmetry extend reflection and rotation into 3 dimensions		**X**
6 Colour Patterns (can be done at a basic level and as a background for symmetry in three dimensions)		**X**
7 Space Fillers		
8 Double edge length tetrahedron		
9 Stella Octangula		
10 Stellated Polyhedra and Duality		
11 Faces and Edges		
12 Angle Deficit		
13 Torus		

Layout

The activity description is in this font, with possible speech or actions as follows:

Suggested instructor speech is shown here with

possible student responses shown here.
'Alternative responses are shown in quotation marks'.

1 Class Safety

First ask the class to gather around an open area on the floor. If necessary, move furniture out of the way to provide space to demonstrate. Have a number of triangles close by and easy to reach.

Talk about safety rules:

> No running, jumping or sliding on the triangles and do not swing them or throw them or lift them above shoulder height. Also keep the corners away from you face and eyes anyone else's face and eyes. Although each corner is fine to hold and they are light, they must be used safely.

2 The equilateral triangle

Now hold one triangle up and ask:

> Does anyone know what kind of triangle this is?

'Equilateral' or 'Equiangular'

Both are correct.

> What do "equi" and "lateral" mean and does anyone know where the words come from?

'Equal lengths' or 'Equal sides'

> The prefix 'equi-' means 'equal' and 'lateral' means 'sides', both from Latin. Also equiangular means having equal angles.
>
> If the three angles all have equal measure, what is the measure of each angle in degrees?

60 degrees

> Why?

They add up to 180

Yes, the angles in a triangle always add up to 180 degrees.

Can anyone prove that the sides are all equal in length?

One way would be to measure; another would be to take a second triangle and lay it on top of the first. Say:

Hmmmm, these are exactly the same size and shape.
What term do we use for this situation?

'Equal' or 'Congruent'

'Congruent' is the correct term.

Then, rotate the top triangle to line up a different side on the lower one, and repeat again. The same side on the top triangle matches exactly with each of the three sides on the other triangle. Also you can directly compare one side with all three of the other triangle as in figure 1.

Figure 1 Comparing side lengths directly

Before connecting triangles, try to have the students visualize and predict the rhombus shape by asking:

If you connect two triangles together along one side
from each triangle, and then lay them out on the floor.
What shape will be formed?

Keep in mind or write down all responses for later verification. Do not affirm any responses until the figure has been made.

3 Tying bows

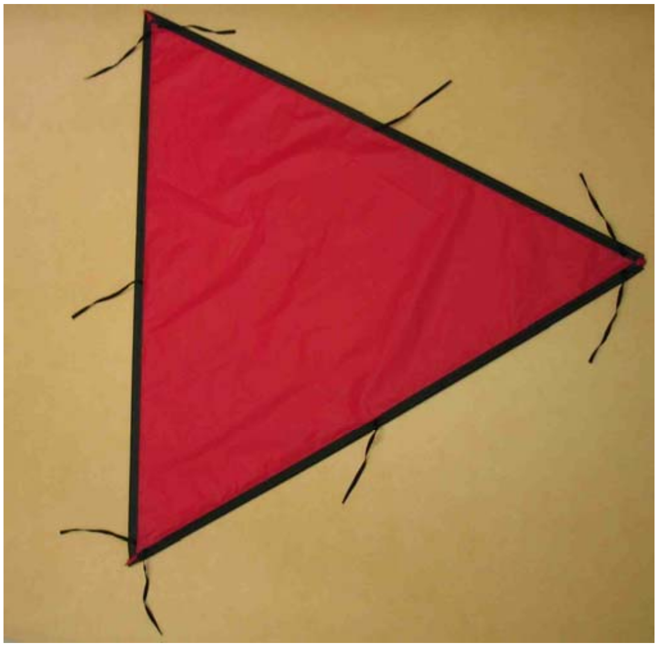

Figure 2 Showing 3 laces on each side

Point out by touching, to show how each side has 3 laces. Demonstrate how to tie two sides together using single shoelace bows. First the middle laces, then near the two ends of the sides.

Just a single shoe lace bow, so it is easy to undo.
No double knots!!

Figure 3 A shoe lace bow

Tell them, perhaps demonstrating (and remind them later):

> Undo the laces by pulling on the loose ends not on the loops, be careful not to make knots.

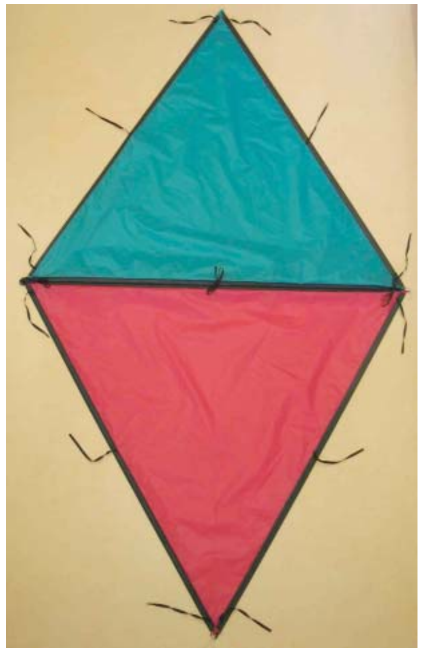

Figure 4 The outside laces left untied

Point out by touching the extra laces at the ends on the outer sides which are left to be tied to other triangles. See figure 4.

> These laces at the end of the joined side are not tied together yet, so that later they can be tied to other triangles.

4 Discussing the rhombus

Open the shape, lay it flat, and discuss:

> What is the name of the new shape?
>
> *'It is a diamond' or 'quadrilateral'*

They may suggest 'diamond' especially if you hold it up from one of its acute angled vertices or if it appears in a vertical orientation on the floor as in figure 4. Doing this provides an opportunity to clarify formal vocabulary and identify some obvious properties. Now affirm that:

"Diamond" is an informal name you may have heard before but the formal name is "rhombus".

Some may argue that a rhombus must lie with one pair of parallel sides horizontally oriented as in many textbooks or printed worksheets. Simply turn the figure and affirm that its name is not dependent on its orientation.

Even if we turn it around at different orientation, it is still the same shape and we still call it a rhombus.

You may wish to consult your curriculum definition of rhombus and verify that this shape meets the requirements. Then instruct the class:

Get into pairs and take two triangles and make a rhombus by tying them together.

Then come up with as many properties of the rhombus as you can find.

'Parallel sides', 'equal sides' or 'symmetric'

The responses may be ambiguous and incomplete compared to the following list properties:

- 4 congruent sides
- 2 pairs of parallel sides
- opposite angles are congruent (one pair 60 degrees; one pair 120 degrees)
- line of reflection symmetry where the triangles connect (figure 5 left)
- another invisible line of reflection symmetry connecting the two vertices with acute angles (shown with added tape in figure 5 right)
- 180 degree, or half turn, rotational symmetry about the centre. (Your curriculum may also call this order 2 rotational symmetry which means a half turn done 2 times make a full turn, getting you back where you started. It also relates to the next property)
- There are 2 orientations of the figure that look the same as it is rotated through a whole turn, therefore features away from the centre come in pairs (groups of 2) that get swapped over by the half turn.

The next section goes into symmetry in more depth.

You can run a length of masking tape between two opposite vertices to show the lines of reflection (figure 5).

Test your masking tape quality in advance or it may tear too easily and be useless in class. Also remember to remove the tape at the end of the lesson as the glue can dry on the triangle and be difficult to remove.

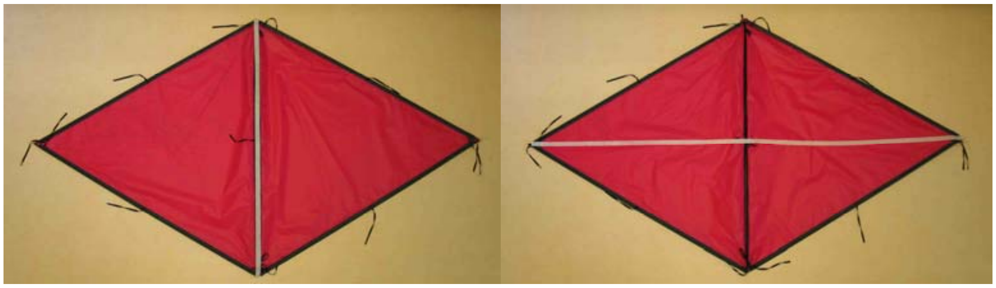

Figure 5 Added tape on lines of symmetry

This makes the two obtuse isosceles triangles apparent in figure 5, right. Pointing all the way round the outline of one of the obtuse triangles ask:

What are the possible names of this type of triangle?

'obtuse', 'isosceles'

Can you see any other triangles?

Right angle

Have a student point all round the outline of one right triangle, then ask:

How many right triangles are there?

4

Pointing to or touching the middle point ask:

What is the angle on each of these triangles at this point where they meet?

'Right', '90'

Yes a right angle is 90 degrees. What is 4 times 90 degrees?

360 degrees

Yes 360 degrees is a whole turn all the way round, so a right angle is ¼ of a whole turn.

5 Comparing 2-dimensional symmetries of the rhombus and regular polygons

This is a good opportunity to explore the 2-dimensional symmetries, reflection and rotation, of the rhombus and regular polygons such as the square and triangle. Tape both lines of reflection on the rhombus as in figure 6.

Figure 6 The 2 lines of reflection and ½ turn rotational symmetry

Ask:

What can you say about the two lines of reflection?

'They are at 90 degrees'
'They are perpendicular'

When two lines meet at 90 degrees we say they a perpendicular.

What is special about where they meet?

'In the middle', 'centre'

Centre of what…?

The shape

What about rotation…

It is the centre of rotation

Yes the lines of reflection meet at the centre of rotation.

What kind of rotational symmetry is there?

½ turn

To help make the rotations clearer you can use two rhombi on the floor. One is placed on top of the other. The one on top can rotate, while the one below is stationary. The stationary rhombus acts as a reference on the floor.

The rhombus on top is in one position where it lines up with the other. How many other positions can you turn the top rhombus to so it lines up again without lifting it off the floor or flipping it over.

1 more

Show us.

Can anyone see any more without lifting the rhombus off the floor or flipping it over?

No

So how many in total?

That is 2

Yes a rhombus has 1/2 turn rotational symmetry and can be rotated to 2 positions that lie within the same outline.

In addition you can do the above for the equilateral triangle as in figure 7.

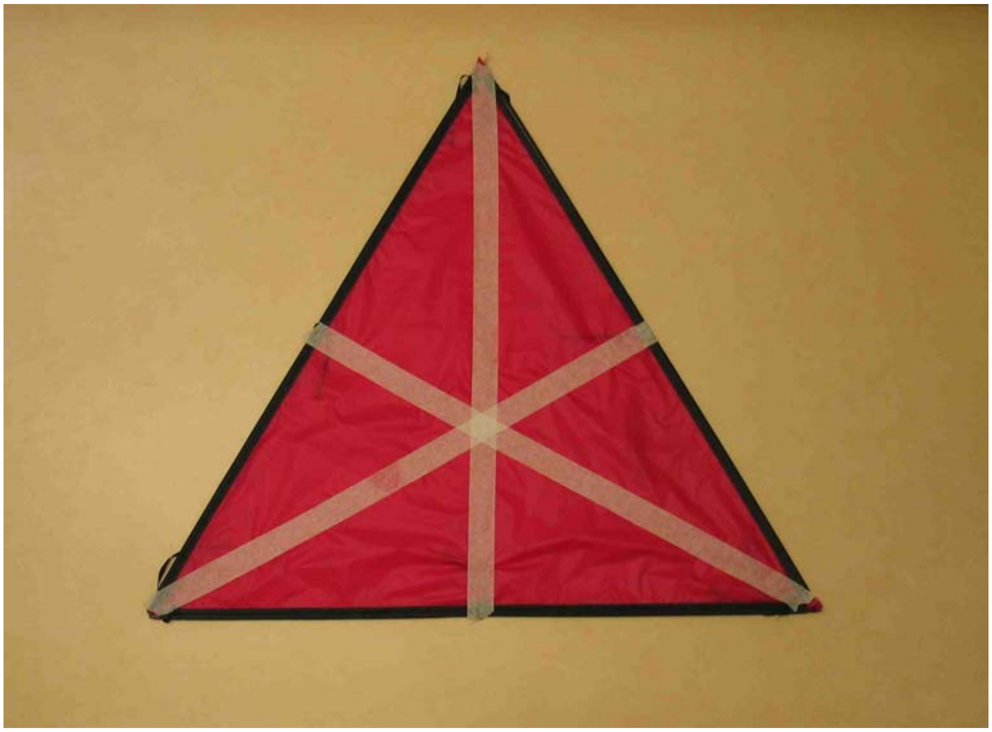

Figure 7 The 3 lines of reflection and 1/3 turn rotational symmetry

Now compare the rhombus ½ turn rotational symmetry with the triangle 1/3 turn rotational symmetry. This can demonstrate the 3 orientations of the figure that look the same as it is rotated through a whole turn. It also demonstrates that features, such as the vertices, away from the centre come in groups of 3, that get cycled through by the 1/3 turn. Also if the 1/3 turn is done 3 times, then there has been a whole turn and everything is back in its original position. This is also revisited in the 'turn and stop game' in activity **5 Symmetry**.

The number of lines of reflection meeting at the centre of rotation can be related to the type of rotational symmetry: 2 lines of reflection correspond to a ½ turn, and 3 lines of reflection correspond to a 1/3 turn.

Note that not all shapes with rotational symmetry have lines of reflection, such as the letter S or Z. Also the letter C demonstrates how one line of reflection alone does not correspond to any rotational symmetry.

Furthermore you can relate angles between lines of reflection and angles of rotational symmetry for the two cases, yielding a ratio of 1 to 2 respectively. A wider range of regular polygons can also be included for a more extensive version of this activity, showing the same pattern of results for squares, pentagons, hexagons etc.

