3 Pyramids

Themes	Terminology, etymology (word origin), pyramids and their 2-D base shapes.
Vocabulary	Base, pyramid, regular, convex, concave, vertex, vertices, side, edge.
Synopsis	Build pyramids with regular bases (triangle, square, pentagon) and compare properties.

Overall structure	**Previous**	**Extension**
1 Use, Safety and the Rhombus	**X**	
2 Strips and Tunnels		
3 Pyramids		
4 Regular Polyhedra (is the direct continuation of this activity)		**X**
5 Symmetry		
6 Colour Patterns (can be done at a basic level and as a background for symmetry in three dimensions)		**X**
7 Space Fillers (use polyhedra at a basic level)		**X**
8 Double edge length tetrahedron		
9 Stella Octangula		
10 Stellated Polyhedra and Duality		
11 Faces and Edges (introduce number patterns and graphing without geometric pre-requisites)		**X**
12 Angle Deficit		
13 Torus		

Layout

The activity description is in this font, with possible speech or actions as follows:

Suggested instructor speech is shown here with

possible student responses shown here.
'Alternative responses are shown in quotation marks'.

1 Three triangles make a trapezoid

Connect two triangles along one side to form a rhombus (as in the Introductory Activity). Lay the rhombus on the floor. Ask:

What figure will be formed if a third triangle is connected to one of the sides of the rhombus?

Trapezoid

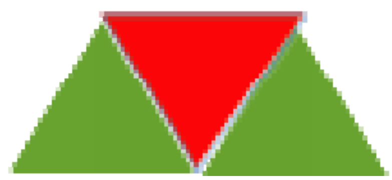

Figure 1 Three triangles make a trapezoid

Then lay the third triangle down. It is not necessary to tie it to the rhombus yet.

What if the third triangle is placed next to a different side?

Still a trapezoid

Why?

'All the sides are the same'
'rhombus is symmetrical'

Demonstrate as on figure 2.

Figure 2 Another way of adding a triangle to a rhombus

2 How many triangles fit around a point?

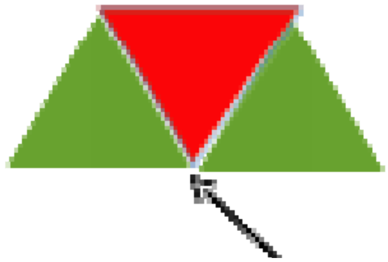

Figure 3 The common point.

Point to the common point where all three triangles meet (figure 3) and ask

> how many triangles would be needed to completely surround this common point where all three triangles meet?

'3', '4', '5', '6'

The purpose is to press them to use mental imagery. For many this is an under-developed skill that will be activated many times at increasing levels of abstraction as the activity progresses. Now lay down three more triangles around this common point as in Figure 4, to form a hexagon.

Figure 4 Six triangles around a point

> How many triangles fit around the point?

6

3 Examining the Hexagon

Ask for the name of the figure

Hexagon

and also what might be "special" about this particular hexagon.

'Same length sides', 'symmetric', 'regular'

Apart from being made from the giant triangles, it has congruent sides and congruent angles.

What term is used for such a polygon?

'equilateral' 'regular'

The usual term is 'regular hexagon' meaning it has 6 congruent sides and 6 congruent angles. It also has symmetry.

You can investigate the symmetries with the class if you wish. It has 3 lines of reflection passing through vertices and three passing through midpoints of edges. So it has 6 lines of reflection and rotational symmetry of order 6.

4 Removing a triangle

Figure 5 Removing a triangle from the common point.

Remove one of the triangles as in figure 5 and ask

Is this new shape regular?

No

It is equilateral?

'Yes' 'No'

It is equilateral because all sides are the same length

Is it equiangular? Are all the angles the same?

No

Go round touching each vertex and verifying students know the angles; 60, 120 and 300 degrees, or at least that they occur in the multiples of 1, 2 and 5 of the 60 degree angle. So the shape is not equiangular.

Is the shape regular then?

No

It is only regular if it is BOTH equilateral and equiangular. So it is NOT regular because it is NOT equiangular.

Also verify that the concave angle is measured as 300 degrees on the inside, not 60 degrees on the outside. Then cover the vocabulary:

What do we call a shape which has at least one vertex pointing inward?

Concave

What do we call a shape which has every vertex pointing outward and none pointing inward?

Convex

The shape is a concave equilateral heptagon as it has seven sides.

Note that an angle between 0 degrees and 90 degrees is called acute, an angle between 90 degrees and 180 degrees is called obtuse and an angle between 180 degrees and 360 degrees is called reflex.

5 Visualization of the closing up process

Before closing the gap to make the pyramid, try to have them visualize by asking these questions. Touch the two edges shown with arrows in figure 4, and ask

What will happen if we connect these two edges to close up the gap?

'It will make a bowl'
'You will get a pentagon'
'It will lift up in the middle'
'Pyramid'

How many sides will the base of the pyramid have?

5

What shape will the base be?

'Chrysler logo' 'Pentagon'

Now without closing up any gaps repeat the above visualization exercise with four and three triangles at the common point to make square and triangular based pyramids respectively as in figure 6.

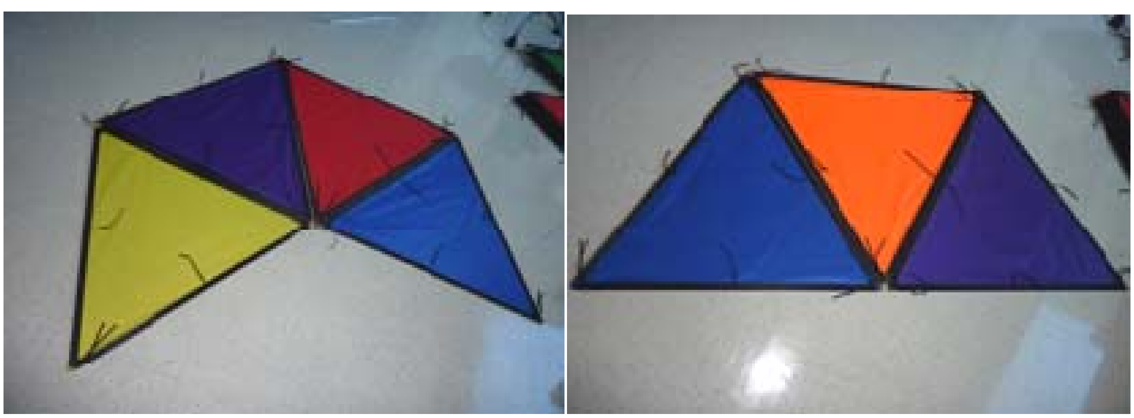

Figure 6 Four and three triangles at the common point.

6 Closing up to make the pyramids

The students now construct the pyramids by tying together the two free edges that meet at the common point shared by all triangles (Figure 7).

Did you get the shape the shapes you expected?

Figure 7 Finished Pyramids

7 Comparing properties

What differences can you see between the shapes?

'Different numbers of sides'
'Tall and short'
'Some are wider'

What are similarities in all the shapes?

'Sloping sides'
'Point at the top'.

Someone point out a side.

So what happens with more sides compared to less sides?

Wider and shorter

And with less sides what do you get?

'Taller and thinner', or 'steeper', or 'pointier'

The above dialogue uses the term 'sides'. Students may be referring to edges or faces. So we now need to discuss the terminology.

8 Edges versus sides

We talk about a triangle or any polygon having sides.
What is the side of a box usually?

A rectangle

When we talk about 3 dimensional objects we talk about faces and edges. Faces can be polygons and edges go along straight lines where faces meet at an angle. We don't use the term 'side' for three dimensional objects because we can't be sure if that is referring to a face or an edge.

9 Computing volumes

For advanced students, it is possible to use trigonometry and Pythagoras to compute the heights of the pyramids. Then use the volume formula V= Ah/3 to get the volume, V from base area, A and height, h.

