Dancing Rope and Braid Into Being: Whole-body Learning in Creating Mathematical/ Architectural Structures

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Abstract

In this interactive, hands-on workshop, the three co-authors/ workshop presenters (with experience in mathematics, math education, weaving and architecture) share insights into the mathematics of braided and twined structures and interests in large scale embodied mathematics learning and community construction techniques. Participants will explore the structures of braid- and rope-making, first through small-scale hand building, and then through collaborative 'danced' construction of twines, braids and Maypole-inspired helical lattices as architectural structures, with the aim of developing deeper understandings of patterns inherent in these patterns. The techniques used in this workshop would provide powerful techniques in classroom explorations of the geometries of ropes and braids for courses in mathematics, teacher education and architecture.

Introduction

Twining (rope-making) and plaiting (braiding) have been important technologies in human cultures since Paleolithic times, and hold a fascination based in their beauty, utility and mathematical structures. The three co-authors and leaders of this workshop have come together with diverse backgrounds, including mathematics education, weaving, dance and architecture, to explore these structures through innovative whole-body, collaborative, dynamic techniques of 'dancing rope and braid into being'. The hands-on activities in this workshop are based on our understanding of the significance of embodied mathematics learning at different scales and a shared desire to make interesting, beautiful and useful structures through the 'physical algorithms' of coordinated dance movements.

Knoll and Gerofsky have been researching embodied mathematical learning through embodied arts at large, medium and small scales and presenting their findings at Bridges for many years [9,10,15,16]. Forren's interests as an architect have centered on geometric patterns, body scale and participatory community understandings of architecture and their interactions with materials and architectural representations (Fig. 1). His designs include sculptural geometric elements of architect-designed spaces [8]. The co-authors bring their diverse expertise to this

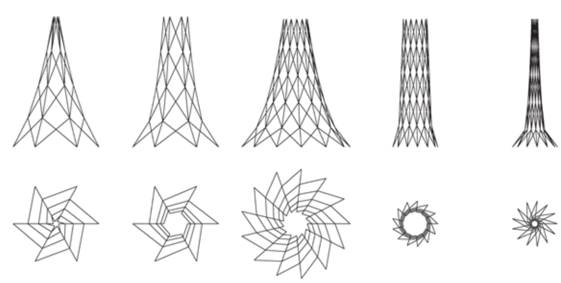

Figure 1: *Architectural braided forms connected to Maypole dance movements* (J. Forren)

workshop.

Bridges participants will 'dance rope and braid into being' in ways that are informed by historical practices, folk dance traditions and principles of embodied learning and body scale, and will create beautiful ropes, braids and architectural structures in the process.

This workshop offers Bridges participants an exemplar of a powerful transdisciplinary, multisensory and multi-scale pedagogy that they might adopt as part of their own teaching practices, at all age levels. It spans the disciplines of mathematics, architecture, history and the arts, bringing together tangible, movement-oriented craft traditions and more abstract mathematical analytic and diagrammatic sense-making skills. The workshop will allow participants to work on a variety of bodily scales, from small, hand-held individual movements to large-scale, whole-body coordinated movements on the group level. Embodied, interdisciplinary, collaborative, inquiry-based approaches to learning are important principles in curricular reform movements for 21st century learners. This workshop will offer participants a fascinating experience of ways to engage these principles in an exploration of mathematical topics.

Pedagogical, historical, mathematical and artistic analysis

Pedagogies integrating different scales and embodied experiences in mathematics learning. This workshop emphasizes the integration of learning in three different modalities and scales: learning about these 3D geometric structures through (a) explanations, diagrams and computer models, (b) making the structures on a small, individual scale, and (c) making the structures on a large, whole-bodied, dynamic, collaborative scale.

This is not atypical of Bridges workshops that might take participants from explanation to small model building to a large 'barn-raising' collaborative effort (using the term coined by Knoll in 1999) [34, 35, 36]. This workshop makes explicit some of the ideas that underlie this way of teaching, draws on educational research into this pedagogy, and justifies an oscillation among these three modalities. We hope this experiential workshop will encourage Bridges participants (and readers of this paper) to consider multi-scaled, embodied approaches in teaching any mathematical topic, with the larger aim of advancing knowledge and praxis in mathematics education.

The most typical and so-called 'traditional' way of teaching mathematics, in schools and universities, is based on teacher lectures, textbook explanations and exercises, student note-taking, homework and tests. This time-honoured approach is seen to be an efficient one, as one instructor can lecture to many students at a time, and much of the onus for sense-making and understanding is left with the students themselves. Students who are able to *imagine* (literally, 'form images of') mathematical patterns from the lecture and connect these images with mathematical notation and logic may be successful in developing an understanding of mathematical ideas. But many lecturers are not expert at creating vivid verbal imagery; not all learners are competent at understanding technical explications without imagery and experiences, and math courses are often infamous for failing high percentages of students. If the aim is to support learning (rather than 'weed out' students), the lecture model may not be very efficient after all.

In contrast, there have been mathematics education studies that show research mathematicians use gestures, imagistic body metaphors, actual and virtual models and objects, and physical movement to develop their own understandings of mathematical concepts and communicate them to colleagues [17, 24, 26]. It would seem sensible that what works for professional mathematicians – an embodied, imagistic, movement oriented approach to mathematics sense-making – ought to be appropriate for students.

However an either/or approach (either sitting still and listening to lectures or engaging in embodied physical experiential learning) is not necessarily a sensible, productive way to bring about positive educational change. We have seen an either/or binary approach in many contemporary educational debates (either phonics or whole language; either memorizing multiplication tables or understanding multiplicative thinking; either always or never using calculators in math class, etc.). Sitting silent and still while listening to teacher lectures may be less than optimal pedagogy for most mathematics learners, but so is a pedagogy that offers only experiential whole-body movement, without any introduction to the rich history and structure of mathematics as a means to interpret those movements. Similarly, it is not helpful to insist on solely small scale (or large scale) mathematical activities, or solely on computer-mediated activities, or alternately, those that do not ever use computers. Any singular approach is far less powerful than a varied repertoire of educational approaches that interact and 'speak to one another', so that learners can begin to make connections and notice resonances among different representations of a particular new mathematical pattern or relationship.

That said, there are particular strengths to large- and small-scale mathematical explorations that are worth noting as educators plan teaching experiences. Thinking about bodily movement is relatively new in mathematics education [11] and for many theorists in this area, all kinds of movement are treated as equivalently 'embodied', from small movements of the hands to locomotion around a space to holistic dance movements. Working in design experiments with mathematics learners, Gerofsky's research group has observed that certain kinds of large-scale movements –those that engage the core of the body and spine, move the body off its centre of gravity and change levels, along with the use of voice and vocalization –clearly promote a higher level of attentiveness and 'noticing' in learners, as humans cannot help but notice the qualities of movements that are (literally) visceral and affect our relationship with gravity and balance [12]. Movements with these visceral qualities are useful educational resources to draw on, but even such potent learning resources must be integrated with other modes of instruction in thoughtfully-designed learning activities drawing learners' attention to mathematically-salient features.

Small-scale embodied movements in mathematical explorations have particular strengths as well. When gestures, movements and physical models are small enough to 'hold in your hand', there can be a sense of control, condensation and conciseness around what is being learned. A small diagram or model can be sensed all at once, and a small gesture can serve as an index to help recall a larger mind-body imagery that may have been developed and experienced at an earlier time. A small or static form of embodiment can serve as a 'memory jog', a cognitive resource to recall and gather mathematical meanings and relationships, and a compactly embodied way to communicate what has been learned, to oneself and others [10, 16].

For educators participating in our workshop, we will draw attention to the oscillation among scales and modalities exemplified in this workshop. While the trajectory of activities will generally move from explanation to small-scale modelling to large-scale whole body movements, there will also be a folding-back and folding-forward to mathematical explorations at each of these scales, with the aim of using whole body, collaborative movement, small scale individual movement and conceptual explanation as cognitive resources that are mutually informative throughout the workshop.

History of rope and braid. Rope-making and braiding are among the most ancient technologies in human history, and have been developed and practiced in most cultures worldwide. Fragments and imprints in clay of ancient ropes and illustrations of rope-making techniques going back as far as 17,000 years ago have been found in Lascaux, France [19], Israel and Egypt [18, 2], Iraq and Greece [5], Central Europe [25], and in the Nordic countries [27], among other places. Most recently, in 2016, archaeologists discovered Paleolithic rope-making tools made from mammoth ivory dated to 40,000 years ago in the Hohle Fels cave in southwestern Germany – the earliest artifact yet showing the ancient provenance of rope-making and twining [3]

Braiding (or plaiting) is distinct from twining (Fig. 2), and has been documented in the Upper Paleolithic in Central Europe, as depicted in the hair/head covering of the well-known Venus of Willendorf and other artifacts dating from approximately 25,000 years ago [25]. A variety of complex techniques of braiding has been shown to be well established 7,000 years ago in China [30], and twined, braided and woven basketry, textiles, garments, hairstyles, ropes and cables and other useful and beautiful technologies have been used in almost every human culture since prehistoric times.

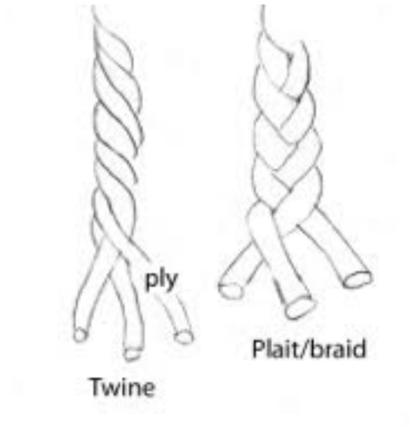

Figure 2: *Twine vs. braid* (E. Knoll)

Traditional Maypole dances originating in Medieval Northern Europe (Germany, England, Sweden) involve a form of communal, dance-based plaiting that was one of the inspirations for this workshop. Explorations of braiding patterns through Maypole dancing as part of mathematics education have been noted by others [21, 28]. Maypole braiding patterns have recently been used as an analogy for modelling technological braiding process as well [14]. The maypole has been a model for braiding machines, with patents for 'Maypole Braiders' issued throughout the twentieth century.

Twining and braiding continue to offer technological challenges and innovations up to the present day, with recent work on creating ropes and spun yarns with materials as small as carbon nanotubes [31] and as large as heavy steel cables for bridges and other architectural structures [29]. There is also new interest in the mathematical and physical analysis of rope structures across times and cultures, from ancient Egyptian rope walks to the rope-like structure of certain kinds of DNA double helices [2, 20].

Mathematical significance of twined and braided structures. The algebraic group structure of braids has been identified since the 1920s, and known in the English-speaking world since the 1940s when foundational theoretical articles were published in English translation [1]. There is continuing contemporary mathematical research on braid group structure and notation [6], in terms of topology and knot theory [23] and cryptography [4]

Twined structures (ropes and cables) are geometrically distinct from braids, and the exploration of their mathematical structure is relatively new. Key studies by Bohr and Olsen [2, 20] connect laid rope (and plied yarn and cable) to the geometry of helices, including questions of chirality, close-packing, zero-twist structures and relative lengths of twined and untwined fibres. Two Bridges 2016 presentations [32, 33] drew on these mathematical bases for an introductory film and workshop on hand-made and Medieval-style machine-made rope, and this workshop builds on and extends the ideas introduced there.

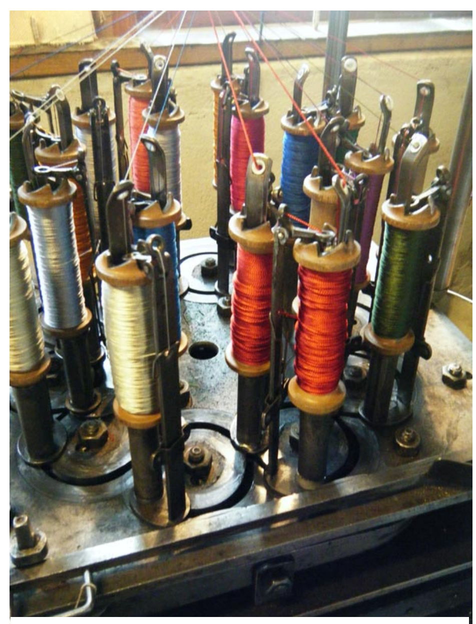

Figure 3: *'Dancing' bobbins on braid machine* (E. Knoll)

Braids and twines have a feature in common that makes them distinct from weaving: where woven structures typically have separate warp and weft threads that meet at right angles, both braids and twines use the same (vertical) strands as both 'bearing' threads (like the warp) and 'weaving' threads (like the weft). This dual functionality of the vertical strands is a structural similarity that might possibly serve as a unifying feature for future mathematical analyses bringing together braid and rope structures.

Why we are interested in 'dancing rope and braid into being'. The workshop leaders discovered a mutual interest in dancing braid and rope into being. Knoll and Gerofsky have experience doing folk dances that involve a braiding motion and discussed ways that dancers could make actual flat or round braid via dance. Both had witnessed 18th-19th century industrial braid machines where the bobbins appeared to be 'dancing' in this way (Fig. 3). They share an interest in exploring mathematical weavings more generally [9, 16, 33].

Forren's interest is in linking digital processes to pre-industrial bodily movements and imagery as part of a demystification of architectural communication and construction processes. His sense of folding metaphoric imagery, bodily movement and abstract architectural ideas in a dynamic oscillation is resonant with Gerofsky and Knoll's educational model of folding bodily movement at different scales with verbal imagery and metaphor, diagrams, and the abstract ideas of mathematics in a dynamic oscillation to promote learning/ understanding.

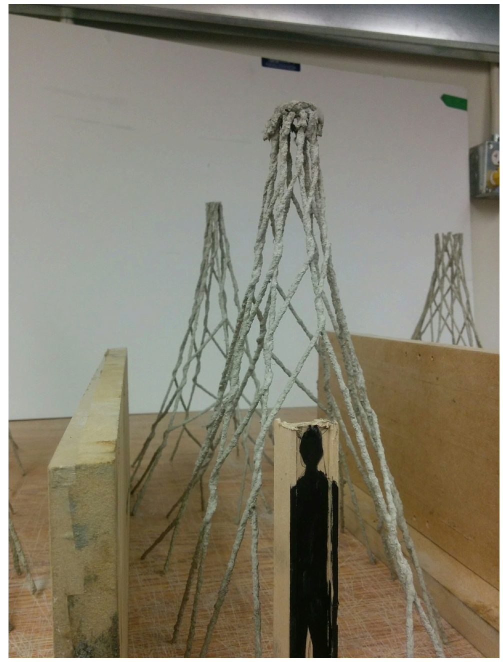

Figure 4: *Braided architectural structures* (J. Forren)

Forren's current research explores prototypes for architectural braidings to create permanent architectural elements and structures of cementitious textiles. These small scale prototypes model a process where twelve-foot tall, rigid helical structures can be shaped through community maypole dances (Fig. 4). This research involves physical and virtual prototyping that explores the parametric potentials of the helical braiding process to create structures of varied shape, proportion, and density.

More information about our work related to this workshop, including videos and photos, can be seen at our website at http://mbodi.teknollogy.com.

Workshop

The workshop will integrate and fold together three elements in making and learning about the structures of rope and braid: (i) small-scale individual making, (ii) large-scale collaborative making through dance, and (iii) discussion and reflection on emergent mathematical observations and ways of noticing.

Introductory talk. The workshop will begin with an introduction to the history and structure of rope and braid, and to elements of their mathematical properties, as outlined in the Analysis section of this paper. Our mathematical focus will be on the geometric analysis of zero-twist helical structures from Bohr & Olsen [2]. Slides, short video clips and handouts will support an initial appreciation of these ancient technologies, and patterns and relationships inherent in their basic structures and variations.

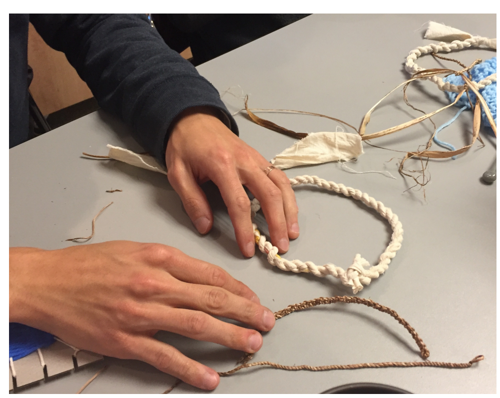

Figure 5: *Making rope by hand* (S. Gerofsky)

Small-scale individual making of ropes and braids. Participants will have the opportunity to try twining and braiding on a small, hand-held individual scale (Fig. 5). Workshop leaders will supply materials and instruct participants in the simple techniques of hand spinning and plying and braid weaving with three, four or more strands. Through individual hand work, we expect that participants will be able to experience the processes in a controlled and holistic way, through the process of individual rope-making in their own hands, at a pace they choose, and with the whole process controlled through each 'maker', without the need to coordinate or collaborate with others. In the process, we expect that participants will gain an experiential overview of the contrasting forms of twine and braid in a way that is different from and complementary to hearing others' explanations or reading information from diagrams.

Large-scale collaborative dancing of ropes and braids into being. Participants will then engage in the main activity of the workshop: 'dancing rope and braid into being' through three collaborative, whole-body group dance/ learning activities. We will ask participants to work together in small groups to derive and test their own, experimental dance steps and movements based on their logical analysis of the structure of rope and braid, and we will also prepare movement processes that we have previously experimented with, to inform our guidance of participants in their inquiry.

• **Rope:** Large-scale rope-making may follow the analogy of the Medieval-style rope making machines presented at Bridges 2016 [32], with participants potentially taking part as spinners, plyers, controllers and time-keepers.

• **Braid:** Large-scale braid making may use moves analogous to those of traditional folk dances. Participants will have the opportunity to view video of bobbins on 18th-19th century braid making machines as an analogy to the moves they might try. The group will experiment with collaborative dance movements that create round and flat braids and make connections with Japanese Kumihimo braid-making.

• **Architectural braid structures:** The final dance activity of the workshop will have participants experimenting with creating architectural elements in the style of traditional Maypole dances. Forren's experience in working with groups using these materials and processes will inform our guidance of participants in learning the dance, and then carrying it out with flexible fibres, on both small and large scales.

The finished braided architectural elements, and the braids and ropes created in the workshop, could be displayed near the Bridges gallery displays, or could be incorporated into Family Day displays and activities as well.

Closing reflections and discussion. In the closing portion of the workshop, participants will have the opportunity to write, draw and discuss the variety of experiences of twined and braided structures they have encountered in the workshop, and to reflect on contrasts and interactions among these different modalities of mathematical/ architectural learning: through listening; through making in small-scale, individual, hand-held ways; and through making in large-scale, collaborative, whole-body ways. Workshop leaders will be attentive to insights gained from participants' reflections to inform ongoing research in mathematical learning via artistic engagement integrating a variety of modalities and scales.

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